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13. ABSTRACT

Bayesian statistics provide the necessary mathematical techniques to pool all available subjective and experimental information when estimating reliability. The uncertainties associated with analytical predictions or limited test data considered separately are significantly reduced when these two sources of information are combined. The introduction of judgment and pertinent engineering theory and experience to qualify point estimates is the key to realistic and practical solutions to decision problems in which reliability is a primary consideration.

A method for periodic reliability assessment is presented. A hypothetical example is used to show how iterative inference on system reliability can be drawn from initial estimates of unit/subsystem reliability and heterogeneous time and failure data accumulated during various stages of design verification, electrical performance, environmental, etc. testing. Sample worksheets for recording inputs required for the assessment technique are provided as an appendix.

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BAYESIAN RELIABILITY ASSESSMENT
FOR SYSTEM PROGRAM DECISIONS

THESIS

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BAYESIAN RELIABILITY ASSESSMENT

FOR SYSTEM PROGRAM DECISIONS

THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

in Partial Fulfillment of the

Requirements for the Degree of

Master of Science

by

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Capt USAF

Graduate Reliability Engineering

December 1971

Preface

This thesis is the presentation of the results of an extensive literature search to discover applications of Bayes formula for the solution of typical decision problems encountered by reliability and maintainability, R/M, engineers.

The purpose of this study is to present an overview of the basic elements and fundamental concepts of general decision theory and show how they might be useful to the R/M practitioner. It is hoped that this brief exposure to some of the organized and systematic techniques to decision-making will create a thirst for more detailed individual investigation for ways to treat similar situations arising in both personal and professional life.

My most sincere thanks are extended to my Thesis Advisor, Professor A. H. Moore for his inspiration, motivation, and consultation. I am also deeply indebted to my wife, [redacted] whose degree of belief in me was great enough to assign a prior subjective probability of thesis completion equal to one. Bayes rule always works with a driving force like that.

Lewis R. White

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Abstract

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BAYESIAN RELIABILITY ASSESSMENT FOR SYSTEM PROGRAM DECISIONS

I. Introduction

The number of cost-plus type government contracts which permit generous budgets for extensive reliability/maintainability studies and tests to minimize risk is dwindling. The time has come when both government and industry have been forced to tighten their money belts. The resulting squeeze has made backbones stiffer and eyes more pointed. In today's austere atmosphere, what used to be accepted techniques and practices are challenged as potential candidates for modification and revision in an attempt to reduce costs.

But although there is less money to spend, the requirements are just as stringent, if not more so. The impact of this current climate is felt throughout all organizational levels in the military-industrial complex. Essentially, practicality and utility are the keynotes that are replacing desirability, feasibility and availability. The reliability/maintainability practitioner must adjust to this new environment.

The theme of getting the most for the money is really nothing new. However, the recent reduction in resources has resulted in more scrutiny of actions taken to achieve this end. No longer is a sole recommendation on a particular issue accepted at face value. The following simple, straightforward and sensible questions are being asked more frequently:

a. Is this the only way to solve the problem?

b. Were other solutions considered?

c. Why is the proposed solution considered the best?

Decision theory provides the framework for systematically addressing these questions and quantifying the uncertainties associated with their answers. The primary purpose of this thesis is to present a method for periodic assessment of system reliability risk based only on analytical predictions and limited usage experience. Particular emphasis is placed on the use of subjective probability and Bayes' formula. These techniques have received much attention in recent years and considerable literature is available for those who wish to pursue more advanced analyses than the one presented. As an aid to those who desire more detail, an additional study objective is to provide a selective listing of references in which general decision theory and Bayesian analyses are used to solve problems of choice usually encountered in the field of reliability and maintainability. These techniques are considered to be valuable analytical tools for assessing alternatives during the establishment, specification, verification and demonstration of quantative reliability/maintainability requirements throughout a system's life.

To achieve study objectives, a comprehensive survey of recent literature was conducted with the aid of abstracting, indexing, and search services. The results of this survey and the subsequent study are presented as follows: Section II summarizes the elements, concepts and notation associated with general decision theory; Section III introduces Bayes' formula and outlines how it may be used to combine

all available relevant information, both subjective judgements and objective data, in the decision process; Section IV addresses the selection of prior distributions applicable to the reliability assessment problem; Section V outlines a proposed method for applying Bayesian techniques to assess the reliability of a system prior to formal testing; Section VI provides a discussion of the assessment technique with a few refinements; and Section VII contains the conclusions and recommendations of the study. A supplementary bibliography is provided as an appendix. Listed are both references which are probably directly applicable to the thesis but were not acquired, and also those which are indirectly or remotely pertinent to the investigation but perhaps of interest to persons working in other functional areas in which the generalized ideas presented are applicable. Also appended are sample worksheets which might be used in situations similar to the example presented.

II. Decision Theory Review

Since man was created, he has been making decisions. As the species expanded, other men (and ofttimes women) started telling him how he should make (or should have made) decisions. In recent years, startling new insights have evolved in the recommended processes for decision-making. Today decision theory is considered almost a distinct scientific discipline. Does this mean that, because we all make decisions (or suffer the consequences from them), we should rush out and obtain a rash of textbooks and read up on the subject? The answer to this question depends on the nature of decisions involved. Most decision situations are trivial and require very little thought; therefore, to perform extensive analyses before making simple choices would be ridiculous to say the least.

Nevertheless, there are occasions when there is much at stake and thorough examination and evaluation of various options and their implications are definitely in order. In these important cases, some knowledge of the precepts and principles of decision theory can be extremely helpful. However, it is stressed from the outset that the study and application of decision theory will not add to the information available to the decision maker, but will merely help him organize the relevant facts and opinions in a manner which will assist him in making a rational choice. Decision theory is essentially a logically consistent systematic approach to the selection of one of several alternative courses of action available to a decision maker.

The methodical approach which is outlined in this section should be recognized as a basic conceptual framework for the analytical treatment of decision situations. It should only be used as a guide and not be considered as a dogmatic delineation of how to treat all problems. Each decision situation is unique and must be handled in light of its particular characteristics. A primary principle in decision theory is that selection problems can be broken down into their constituent elements which are usually easier to analyze. The methodology also puts these individual elements in proper perspective with each other so, although they can be treated separately, the "big picture" is not lost. The detailed analysis of these interrelated portions of the problem are accomplished in various ways but each has a common prerequisite - the willingness to think quantitatively.

This requirement is established so the powerful mathematical techniques associated with decision theory can be employed to formulate and solve decision models. No longer does the decision maker have to rely solely on emotion and intuition. However, the use of analytical tools by no means eliminates or diminishes the importance of a decision maker's experience and his instincts and insights derived from similar situations. Rather than reducing the need for human judgement, the framework actually provides a mechanism for the explicit consideration of personal experience and opinion. As a matter of fact, the additional clarity resulting from the distinct differentiation between judgements involving the likelihood of an event and those concerning the worth of an alternative is a definite advantage of the process.

There are really several processes associated with the general study of decision theory. There is no agreed standard procedure on how to structure and solve decision problems. The synopsis which follows is a presentation of a process that is typical to the solution of decision problems dealing with uncertainty. The following parts, principles and procedures are considered of the most value for the decision maker who works primarily in the reliability and maintainability arena.

Basic Elements

As previously mentioned, decision situations can be broken down or decomposed into sets of factors which can be analyzed individually at first and collectively later. Of course it is more difficult to disect and recombine partial analyses of problems which are more complex or have greater uncertainty surrounding them. Although the decomposition effort may be minimal or extensive, there are several sets of factors that are common to all decisions made under uncertainty. Depending on the preliminary results of a partial analyses, this common or primary group may be augmented by an additional or supplementary group.

Primary Sets. The primary group of elements consists of: a set of the available alternative courses of action; a set of the possible states of nature; and a set of consequences of each alternative and state of nature combination.

The action set will be denoted by A,

where $A = \{a_i\}$ for $i = 1,2,\ldots,m$

and a_i = a specific course of action

m = the total number of alternatives which are feasible and practical

There is no prescribed procedure on how to develop a list of possible actions that might be available. Obviously, the compilation of this list deserves considerable thought to assure all reasonable options are included. The decision maker is encouraged to apply his ingenuity, imagination, and initiative to the fullest extent. The methodology does require that members of the action set be mutually exclusive and collectively exhaustive. The mutually exclusive property limits the selection to only one member from the set - combinations are not permitted. The collectively exhaustive property merely means that the list should be complete in the sense that one of the members must be chosen. Thus, the solution to the decision situation is the selection of a single item from this action set. The primary difficulty in making this choice is usually due to the uncertainties of the situation, in other words, not knowing exactly what will happen should a particular alternative be selected. These uncertainties usually stem from unknown states of nature which constitute the second primary set. The nature state set will be denoted by Θ,

where $\Theta = \{\theta_j\}$ for $j = 1,2,\ldots,n$

and θ_j = a possible event that can occur or happen which is relevant to one or more of the actions, a_i, under consideration. Nature exists in exactly one of these unknown states.

n = the total number of states that have a potential impact on the problem.

As in the action set, there is no suggested way to enumerate all the uncertainties involved in a particular decision situation. Again, the skills of the individual decision maker play an important role in judging the relevance of the many unknown factors bearing on a particular problem. The decision maker usually has considerable latitude in assigning members to the nature state set. However, he must obey the exclusive and exhaustive rules previously mentioned. Therefore, he should clearly and concisely describe each event in the set in a manner which prevents overlapping and assures completeness. Difficulties encountered in following the exclusive rule can be alleviated by judicious definition and careful grouping of each member. In many situations, it may be necessary to construct the set by taking pairs, triplets and higher order combinations from other lists of more obvious factors. This technique is also sometimes helpful in adhering to the exhaustive rule. Of course, it is highly unlikely that each possible uncertain event can be identified in each and every case. The time and effort devoted to the composition of the nature state should be tailored to the importance of the decision required. Essentially, the practical significance of the exhaustive rule is that the list of uncertain events should cover all the known contingencies likely to affect the selection of a particular course of action. This is not to say that there might not be unknown factors that bear on the decision problem. Thus, there will be a few cases when the consequence of a particular action will depend not only on factors considered by the decision maker but also unknown unknowns.

However, in general, consequences can be foreseen and it is convenient to designate them as a set.

The third and final primary set, the consequent set, will be denoted by C,

where $C = \{C_{ij}\}$ for $i = 1,2,\ldots,m$
$j = 1,2,\ldots,n$

and C_{ij} = the result of a particular available course of action, a_i, if a specific anticipated state of nature, θ_j, occurs.

A consequence is essentially the position, posture, predicament or state of affairs associated with an individual optional act and uncertain event combination. Thus, conceptually, there are $m \cdot n$ consequences possible in every decision situation. A particular course of action, if selected, could lead to one of n consequences depending on which state of nature exists. Within this group, there are consequences which are good, desirable, beneficial, or profitable as well as those which are bad, unwanted, detrimental or costly. The final choice should represent an optimum balance among these positive and negative features. Sometimes this choice is obvious. Many times the selection requires no more than just considerable deliberation of the facts on hand. But there are some instances when assessed risks are too great and additional data must be obtained and analyzed before an effective discrimination among the alternatives can be made. When these situations occur it is necessary to extend the basic approach to include the formulation and evaluation of supplementary sets.

Supplementary Sets. The supplementary group consists of two sets - one which includes the various ways to obtain additional information

and one which includes the supplementary data accumulated.

The first supplementary set is really a family of possible experiments. This set will be noted by E,

where $E = \{e_k\}$ for $k = 1,2,\ldots,r$

and e_k = a specific experiment

r = the total number of experiments which are applicable and appropriate.

Members of this set consist of any data collection methods, techniques, plans, etc., which can be used to discover more about the true state of nature. A working knowledge of statistical theory is a definite asset in the formulation and application of this set. The manner and extent to which sampling is accomplished directly affect the validity and creditability of the results. Usually, both the type of additional information needed and the methods of acquisition are obvious. The need for the data collected to be both representative and sufficient is intuitive to most people. However, if experimentation is to be an important aspect of a complex decision problem, then the advice of a competent statistician is almost mandatory. His role will be to assure that the experiments being considered can generate useful outcomes. Whether personally observed or generated by a sophisticated experiment under someone else's direction, the resultant sample data constitutes the other supplementary set.

The outcome set will be denoted by X,

where $X = \{x_l\}$ for $l = 1,2,\ldots,s$

and x_l = a possible outcome of the experiment set E

s = the total number of potential observations of all e_k in E.

The size of this set can vary from a few denumerable elements for simple decision situations involving discrete variables to an infinite list of items for complex problems involving continuous variables. In most cases, the enumeration of all possible outcomes would probably require an inordinate amount of time and effort. Generally, the primary interest is to obtain an appreciation for the range of outcomes. For routine, uncomplicated decision situations, the lowest and highest values anticipated for the results of a small number of experiments are usually either apparent or easily obtained. However, in more complex situations, the assistance of a statistician is highly desirable. Both the range and other meaningful characteristics of the expected results can usually be determined from mathematical equations, tables, charts, and other statistical tools not understood by all decision makers. One such tool is Bayes Theorem which allows the pooling of all available information when making inferences about unknown quantity (i.e., the true state of nature θ). This Theorem, and the related prior distribution (which reflects knowledge before experimentation) will be addressed in more detail in the next two sections. The importance of the roles that these topics have in the overall decision process will become apparent shortly.

Fundamental Concepts

In the preceding discussion, a general description of the essential and auxiliary ingredients was provided along with some of the conditions and constraints for their formulation. Now, general guidelines will be outlined on how to shape these basic building blocks and construct a fundamental framework within which a variety of decision problems can be solved.

Definition of the Problem. First and foremost in any problem solution process should be a careful examination of the issue or question raised. Odiorne defines a problem as "the difference between present condition and desired consition" (Ref 154:15). He also states that commitment is necessary for problem solving for "the committed man has to choose and decide among alternate solutions and moves. The uncommitted man can delay, put it off, and not get things done." (Ref 154:16). These "things to do" constitute the desired condition or objectives. Objectives should be stated in clear concise terms which collectively can be a guiding light providing illuminating direction toward their achievement. The precise and complete delineation of the difference between what is wanted and what is available is a primary prerequisite for future efforts to bridge the gap by enumerating and evaluating the alternate avenues available.

Assessment of the Uncertainties. The impact of each of these options is dependent on an unknown state of nature existing at the time action is taken. Although the exact state is unknown, it is assumed to be a member of the set, Θ, of possible states. In most situations, the members of Θ are not all equally likely to occur. In fact, the decision maker may have encountered similar situations and thus has a basis for weighing a particular θ_j more heavily than others.

> The English language has several words to describe aspects of the uncertainty that is felt on such occasions: one of them, likely, has been used in the previous sentence. Others are probable, credible, plausible and expressions derived from words like chance or odds. Our aim is to describe this uncertainty numerically; for number is the essence of the scientific method and it is by measuring things that we know them. (Ref 137:13).

The term which will be used throughout this thesis is probability. It is intended that a generic definition apply to this yardstick for measuring uncertainty. Subsequent to the following discussion on the various kinds of probability, the word will be used without qualifying adjectives.

The word "probability" is used by practicing mathematicians and statisticians in several different ways and means different things to different people. To circumvent these semantic obstacles, various descriptors have been used to provide more explicit definitions. Typical of the phrases often used are those discussed by Good:

> ...a physical probability (also called "material probability," "intrinsic probability," "propensity," or "chance") is a probability that is regarded as an intrinsic property of the material world, existing irrespective of minds and logic... A psychological probability is a degree of belief or intensity of conviction that is used for betting purposes, for making decisions, or for any other purpose, not necessarily after mature consideration and not necessarily with any attempt at "consistency" with one's other opinions... When a person or persons, called "you," uses a fairly consistent set of probabilities, they are called subjective ("personal") or multi subjective ("multi personal") probabilities. (Ref 94:6).

Thus, physical probability corresponds to the relative frequency interpretation, and is measured by observing how often a particular event occurs in relation to the total number of attempts made. Subjective probability (the special case of psychological probabilities to which future discussion will be limited) applies when "one is quantifying his personal judgements based on his experience and knowledge, insight and information." (Ref 116: 28). The mathematical properties of both these general classes must obey the postulates and laws of probability theory. Using the event set notation and defining P( ) as "the probability of the event ( ), the three basic requirements for

mutually exclusive (or disjoint) events are as follows:

(1) $0 \leq P(\theta_j) \leq 1$

(2) $P(\Theta) = 1$

(3a) $P(\theta_1 \text{ or } \theta_2 \text{ or } \ldots) = P(\theta_1) + P(\theta_2) + \ldots$

(3b) $P(\theta_j \text{ and any other } \theta_j) = 0$

Rule (1) represents the only new requirement since rules (2) and (3a, 3b) are merely reexpressions of the exhaustive and exclusive rules, respectively. Rule (1) can also be satisfied if odds are quoted for uncertainty. For example, if a particular event is favored 4 to 1, the equivalent statement is that it is likely to occur 4 out of 5 times or with .80 probability. Another aid in establishing a number between 0 and 1 as a measure of likelihood is the comparison of the chances of a particular event with those of a random point falling within a designated area of a unit square. (Ref 137:19).

<u>Comparison of Consequences</u>. In addition to evaluating the data (either objective or subjective) relating to the likelihood of the state of nature, $P(\theta_j)$, the decision maker must also examine the potential impacts of the individual choices, a_i. As previously stated, not all consequences are equally favorable. Thus, the decision maker should list the possible consequences in a relative ranking sequence which reflects the degrees of achievement toward the objectives. Obviously, the particular consequence which represents the greatest step toward the goal is to be the most preferred. Also, great gains should rank higher than shorter ones. Therefore, as Lindley explains:

> It follows that the next task is to provide something more than just a ranking of the consequences. In order to do this a standard is introduced and coherent comparison with it provides a numerical assessment, just as with the uncertain events. In the case of probability, the standard was a random point in a unit square. For the consequences, two reference consequences are used; one of these is better than, or at rate not worse than, any of the consequences in the relevant table; the other is similarly worse than, or at most not better than, all the C_{ij}. (Ref 137:52).

He goes on to develop a separate probability measure relating to the attractiveness of a particular C_{ij} in comparison with the most preferred consequence. He calls this numerical measure "a utility of the consequence." (Ref 137:53).

Characteristics of Utility. The numerical measure for consequences does not have to be a probability function. In fact, a quantitative yardstick, although highly desirable, is not mandatory. Qualitative expressions may be used if more appropriate. Miller states that "in addition to the common practice of measuring the utility of consequences on an objective scale such as dollars, gain may be a more subjective concept including factors such as reputation, happiness, security, or any other characteristic associated with well-being." (Ref 148:376). In addition, Schaifer indicates that "consequences might also involve "physical assets, technological know-how,...the behavior of various people,...and the decision maker's own personal position..." (Ref 186:40). The point really is that there is no common denominator or standard dimension against which the worth of a consequence can be gauged. However, there is a definite advantage to using numerical measures because they are easier to handle mathematically. The basic requirement in assigning a quantitative index to a value judgement is

coherence. A decision maker is considered coherent if he uses a utility function which assigns a higher utility number to a most preferred consequence and equal numbers to consequences for which he is indifferent. A particular quantity for a utility measure may be designated $U(C_{ij})$ and may fall within any arbitrarily selected range that is appropriate to the situation.

Calculation of Expected Value. Once the numerical measures $P(\theta_j)$ and $U(C_{ij})$ have been determined, the next step is to combine them in a manner that the relative merits of the various a_i can be assessed. To do this, the quantity known as expected value will be used. The expected value, or mathematical expectation, of a particular a_i, written $E(a_i)$ can be computed from the following relationship.

$$E(a_i) = \sum_{j=1}^{n} U(C_{ij})P(\theta_j) \quad . \tag{2-1}$$

Thus, $E(a_i)$ is a measure of the extent that a_i can solve the problem considering the circumstances known to the decision maker. The logical choice is to select the alternative from A which has the greatest $E(a_i)$. Since this choice is highly dependent on the $P(\theta_j)$, these quantities must reflect all known information about Θ - not necessarily just the recorded results of sampling. Bayes Theorem serves as the basis for the current practice of combining subjective and test data.

III. Bayes Theorem

Bayes Theorem is essentially a simple relation between probabilities that two different events will occur. The basic expression which describes the relationship is

$$P(A/B) = \frac{P(A)\, P(B/A)}{P(B)} \tag{3-1}$$

where the / is to be read "given."

Development

There is really nothing special about the formula per se - its derivation is quite straightforward.

Consider the "probability diagram" depicted in Figure 1, where A and B represent two events (not disjoint as were the θ_j).

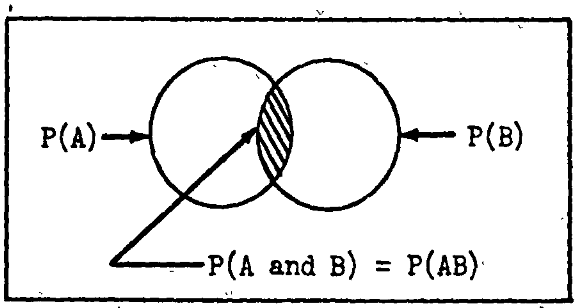

Figure 1

Probability Diagram
(from Ref 213:344)

The probability of event B given that event A has occurred, P(B/A) is the portion that common shaded area, P(AB), is to the total area P(A). In equation form,

$$P(B/A) = \frac{P(AB)}{P(A)} \tag{3-2}$$

Likewise, $P(A/B)$ can be determined by

$$P(A/B) = \frac{P(AB)}{P(B)} \tag{3-3}$$

Note $P(AB)$ is common to both equations and thus

$$P(AB) = P(A/B)\ P(B) = P(B/A)\ P(A)$$

When each term of the latter equality is divided by $P(B)$, Bayes Theorem results.

$$P(A/B) = \frac{P(B/A)\ P(A)}{P(B)} \tag{3-4}$$

<u>Interpretation</u>

So what is all the fuss about an easy rearrangement of terms? The answer is in how these terms may be interpreted. Addressing each term individually and replacing A with θ_j ; and B with x_1:

$P(\theta_j)$ = Prior probability (what is known about θ_j before additional data x_1 is available)

$P(x_1/\theta_j)$ = Likelihood of observing x_1 (how probable is the sampling results, assuming the true state of nature is a particular θ_j)

$P(x_1)$ = Probability of test observations for any θ_j in Θ

$P(\theta_j/x)$ = Posterior probability (what is known about θ_j after x_1 has been obtained and analyzed)

Thus, Bayes Theorem provides the framework which allows redetermination of $P(\theta_j)$ based on additional information. Decisions dependent on knowledge of $P(\theta_j)$ are better when based on all available information. But there is some controversy between the Bayes approach and classical statistical methods when some or all of the prior knowledge consists

of theoretical considerations, design analyses, engineering judgement, etc.

Controversy of Usage

The key to the question concerning the applicability of Bayes Theorem for updating $P(\theta_j)$ is the use of subjective rather than objective information to establish the prior probability. The argument advanced by the purists is that the introduction of intuition and guess-work into statistics constitutes an unnecessary bias and the resultant inferences made are not valid. The reason for these doubts is centered around the various definitions of probability discussed in the previous section. Essentially, there are two schools of thought which are prevalent today. For future discussions, the terms designated by Weir for the advocates of these two philosophies will be used:

(1) "Classicist" which will refer to the group which adopts the frequency interpretation of probability. This faction believes in the concept of unknown parameters located at one unknown point, which can be estimated with increasing precision as test data builds up; information of a non frequency nature cannot be directly used in this estimate. Probabilities based upon outcomes of games of chance (e.g., flipping coins, tossing dice, dealing cards) are of course subject to a frequency interpretation. The ratio of successes to total trials over the very long term can be nearly equated to the probability of the event.

(2) "Subjectivist" which will refer to the group which while dealing with the frequency application of probability to games of chance and other applicable situations, also believes that probability theory can be applied to questions of degree of belief in propositions (e.g., the probability that there is life on Mars). This faction believes it is sensible to talk of probability distributions of parameters based upon degree of belief in the location of the parameter; this permits both frequency and non frequency information to be combined using the central framework of Bayes Theorem. (Ref 213:345).

It is not the intent of this work to delve into the controversy but merely point out its existence. A rather comprehensive discussion of the pros and cons of each of the approaches is presented by Hahn, in the proceedings of the 1965 General Electric Seminar (Ref 214:Sec VIII).

In spite of the controversy, Bayes Theorem and the more inclusive field of what is sometimes referred to as "Bayesian Statistics" have been described as an important step forward in removing some of the constraints of classical theory (Ref 178). Some authors point out that "Bayesian techniques complement classical, statistical methods rather than replacing them." (Ref 74:5). This is especially true when the techniques are applied to reliability estimation and assessment problems.

<u>Application to Reliability Problems</u>

Schulhof and Lindstrom have stated three primary factors which have generated the need to replace time honored classical methods with more modern schemes:

1. Products are becoming more complex and correspondingly expensive.
2. Time is at a great premium.
3. High reliability is being both demanded and achieved. (Ref 187:684).

The paradox is one of greater requirements but fewer resources to verify achievement. The "Classicist" by ignoring prior subjective estimates on the range of the failure rate of an item in essence makes the implicit assumption that the item may possess <u>any</u> failure rate (e.g., $0 < \lambda < \infty$) (Ref 153:3). This hypothesis is extremely costly from the standpoint of the inordinate amount of testing required to narrow in on the true value for a reasonable degree of assurance.

The Bayesian approach recognizes the value of past experience and admits any data - theoretical limits, quantified beliefs, intuitions, whatever - into the analysis. This prior information reduces the scope of sampling to a finite range with resultant savings in time and money. Iterative analyses may be performed to provide periodic updates concerning a system's reliability.

> Bayesian methods can use knowledge gained from development testing to indicate the reliability of equipment at each stage of its development. Tests may also be conducted during design development for purposes other than reliability estimation with full expectation that the data can be integrated into a reliability estimation procedure with reasonable statistical validity. It is possible to ascertain by these methods whether a specified system reliability requirement has a reasonably good chance of being achieved. Conversely, these methods will show whether special effort, such as redesign or modification, is warranted, by indicating the existence of a low probability of design reliability achievement.
>
> By having this objective information available concurrently with each phase in design development, the designer is enabled to make necessary improvements at an earlier state. This makes revisions more compatible with costs, schedules, tool design, and other important factors, and makes achievement of the contractual reliability requirement more certain. As a consequence, the number of tests required to achieve and demonstrate a reliable design can be reduced, resulting in time and cost savings. (Ref 74:5).

In summary, the following are considered practical advantages of using Bayes techniques for estimating reliability:

1. The estimates take both predictions and test data into account.

2. The estimates give reasonable results for little or no test data.

3. The estimates agree with the classical estimates for large samples. (Ref 187:684).

Of course, there are drawbacks to using Bayesian techniques. In addition to the issues raised by the "Classicists," there is the obvious problem of erroneous initial information. What happens if incorrect assumptions are made or faulty logic was used to formulate a considered opinion? How accurate can an overall final estimate or assessment be if based on imprecise inputs? Fortunately, Bayesian methods have an inherent corrective feature since as the quantity of subsequent test data increases, the initial estimates become overshadowed. If desired, the effect of incorrect prior information can be reduced to any stated level by sufficient additional testing. As previously stated, Bayesian estimates agree with "Classicist" estimates after an extensive amount of test and operational life data has been obtained.

The question which must be answered now is "How does one quantify his subjective appraisal of an item's reliability in a manner that is usable?" This is accomplished by using available reliability predictions and related statements of uncertainty surrounding them. Basically, the parameters of the applicable system failure density function are considered to be random variables. A probability distribution function for a parameter is established based on the point estimate and expected range of the predicted value. This parameter distribution is known as the prior distribution and is subject to modification as test data becomes available. Typical prior distributions encountered in reliability and instructions on how to generate them are the subject of the next section.

IV. Prior Distributions

The heart of a Bayesian statistical inference is that a probability distribution is assumed to exist for the unknown true state of nature θ. This prior distribution essentially reflects the amount of knowledge, or degree of belief, before the results of experimentation are available. If absolutely no information is known, all values of θ are equally likely, and logic dictates a Uniform prior distribution. However, in virtually all reliability problems, there exists a significant amount of information from generic or similar parts documented in various handbooks. Also, according to Gottfried and Weiss, "experience indicates that the failure rate of any device is bounded on the left (non-negative) and skewed to the right - larger failure rates than expected are less surprising than smaller values." (Ref 100:603). Although there is no widely agreed rules on how to select a suitable prior distribution, Babillis and Smith have established what is considered to be minimal criteria:

> 1. The prior distribution must adequately reflect what is actually known before test data becomes available. That is, the distribution must be consistent with the available prior knowledge of a component's reliability.
>
> 2. The prior distribution should not imply any assumptions about unknown information concerning the reliability of a component. In other words it should remain as maximally noncommittal and as unprejudiced as possible concerning things which are considered unknown.

3. The prior distribution should not lead to absurd conclusions concerning the component's reliability when it is modified by the data. The selected apriori distribution should not lead to results which are inconsistent with what is known or intuitively felt...However, it is important to realize that these inconsistencies may only be evaluated in terms of what was originally thought to be known. If pertinent information is withheld in establishing the prior distribution, this same information may not be used to discredit the resulting posterior distribution.

4. The resulting equations should be tractable by available mathematical methods. This is purely a pragmatical consideration based solely on the desire for an answer. Through this same crack in the otherwise logical framework also creeps a certain element of empiricism which requires drawing upon experience to get the technique started. (Ref 6:357).

Although the above criteria are general, they do serve the purpose of narrowing the selection process. Perhaps someday simple algorithms for establishing priors will exist. As of this writing there is considerable effort underway to find ways and means for formulating probability distributions for reliability indices. For example, Rome Air Development Center has a current study contract with Hughes Aircraft Company to develop methods for fitting prior distributions to empirical data and combining priors from similar but not identical equipment. The reported results of initial efforts are quite promising. It has been concluded that it is entirely feasible to fit prior distributions to equipment level Mean Time Between Failure, MTBFs, although the amount of data currently in existence to do so is somewhat limited. It was also determined that the probability of MTBF is usually well described by an Inverted Gamma Distribution when the assumed equipment failure time is exponentially distributed. (Ref 182).

Since MTBF or its reciprocal, the failure rate λ, are the most

commonly used measures for time dependent reliability, one merely has to determine which member of the family of Gamma type distributions is applicable to the situation. How does one translate his knowledge concerning the most likely single value and range of uncertainty for MTBF or λ into a Gamma distribution?

Assigning Gamma Parameters

The Gamma family of probability distributions meets all the previously stated criteria. In the most common form, it is described by two parameters and is very flexible. Also, the selected parameters combine readily with Poisson sampling statistics (the Poisson process is an experiment which observes f failures in t time) which is a prerequisite to meet the fourth criterion for priors. If a Gamma prior is updated with Poisson data the resultant posterior is also a Gamma distribution. This property is very convenient for calculation purposes.

Now that the suitability of a Gamma form has been somewhat justified, an explanation on how it fits into the overall decision making procedure is in order. If an equipment failure rate λ is assumed to be a random variable, then

$$P(\theta) = g(\lambda; a, b) = \frac{b^a \lambda^{a-1} e^{-b\lambda}}{\Gamma(a)} \tag{4-1}$$

for $a, b, \lambda > 0$

The probability of a particular outcome of the Poisson experiment ($x = f$ observed failures in t time), given the true state of nature ($\theta = \lambda$) can be expressed as

$$P(x/\lambda) = p(f/t, \lambda) = \frac{(\lambda t)^f e^{-\lambda t}}{f!} \tag{4-2}$$

The remaining term, $P(x)$, of the Bayes formula can be determined in the following manner

$$P(x) = \int P(x/\theta)P(\theta)d\theta$$

$$= \int_0^\infty p(f/t,\lambda)g(\lambda;a,b)d\lambda \tag{4-3}$$

Thus, from the Bayes relationship (3-1)

$$P(\theta/x) = P(\lambda/f,t) = \frac{\dfrac{(\lambda t)^f \; e^{-\lambda t}}{f!} \dfrac{b^a \; \lambda^{a-1} \; e^{-b\lambda}}{\Gamma(a)}}{\displaystyle\int_0^\infty \frac{(\lambda t)^f \; e^{-\lambda t} \; b^a \; \lambda^{a-1} \; e^{-b\lambda} \, d\lambda}{f! \qquad \Gamma(a)}} \tag{4-4}$$

Now that this specific case has been defined, the general expressions involving θ and x will no longer be used, and the expression for the posterior distribution can be reduced to

$$P(\lambda/f,t) = \frac{\lambda^{a+f-1} \; e^{-\lambda(b+t)}}{\displaystyle\int_0^\infty \lambda^{a+f-1} \; e^{-\lambda(b+t)} d\lambda} \tag{4-5}$$

by letting $y = \lambda(b + t)$ the denominator becomes

$$\int_0^\infty \left(\frac{y}{b + t}\right)^{a+f-1} e^{-y} \frac{dy}{b + t}$$

which can be solved by rearranging terms,

$$\left(\frac{1}{b + t}\right)^{a+f} \int_0^\infty y^{a+f-1} e^{-y} dy = \left(\frac{1}{b + t}\right)^{a+f} \Gamma(a + f) \tag{4-6}$$

Substituting this result in equation (4-5)

$$P(\lambda/f,t) = \frac{(b + t)^{a+f} \; \lambda^{a+f-1} \; e^{-\lambda(b+t)}}{\Gamma(a + f)}$$

$$= g(\lambda;a + f, b + t) \tag{4-7}$$

With this proof, a better appreciation for the interpretation of the meaning of the parameters a and b is possible. Since a and f are equivalent quantities, the former can be considered what has been termed as a "pseudo-failure"; likewise, b can be thought of as "pseudo-time" (Ref 214:5-6). So by assigning values for a and b, one may assert his extent of subjective confidence in predicted equipment failure rates. This may be done by examining the mean and variance of the Gamma form defined in equation (4-1),

$$\mu = \frac{a}{b}$$

$$\sigma^2 = \frac{a}{b^2} = \frac{\mu}{b} \tag{4-8}$$

If the predicted value λ_p is equated to the mean, then

$$\lambda_p = \frac{a}{b}$$

$$\sigma^2 = \frac{\lambda_p}{b} \tag{4-9}$$

Thus, by assigning a and b, one is really stating his belief concerning the number of "pseudo-failures" and the amount of "pseudo-time" reflected in the predicted value. For example, if $\lambda_p = 50 \times 10^{-6}$, then the following combinations are possible and are listed in order of increasing certainty from left to right:

$$\frac{a}{b}: \frac{0.1}{2000}, \frac{0.5}{10{,}000}, \frac{1}{20{,}000}, \frac{5}{100{,}000}, \frac{10}{200{,}000}$$

The importance of carefully selecting values for a and b that realistically correspond to prior knowledge cannot be overemphasized. Understatements are costly because of extensive sampling required to narrow the resultant wide range (i.e., large σ^2). Conversely,

overstatements are ill advised because of the significant contribution that the prior will have on the posterior and thus the decision to be made. With these cautions, three methods will now be discussed which address how to select values for a and b.

Upper Bound Method. Two known bits of information must be provided if two unknown quantities (in this case a and b) are to be determined. The predicted failure rate λ_p represents only half of the required input. This value must be combined with another statement relating to additional prior knowledge about the failure rate. One way to encode a conviction of belief concerning a reliability prediction is to assign a probability, ρ, that the true failure rate is no greater than a particular upper limit λ_u. Once this is done, then λ_p can be considered to be either the mean or median value of the prior distribution and published tables or graphs can be used to select values for a and b which satisfy the probability statement. (Ref 214:A2-37). The value a is the same for any λ_p and λ_u combination with an identical discrimination ratio λ_u/λ_p and extent of certainty, ρ. It was found convenient for later use to tabulate an assortment of what is considered typical λ_u/λ_p and ρ. The values for a for discrimination ratios from 1.50 to 10 and ρ from .67 to .99 are given for λ_p equal to the mean in Table 1 and λ_p equal the median in Table II.

Coefficient of Variation Method. Another way to describe uncertainty associated with a prediction is to state the extent of possible inaccuracy in terms of an estimated standard deviation. If an engineer states "I think λ_p is the true failure rate but I could

Table I

Values for a if λ_p Assigned Median

$\psi = \frac{\lambda_U}{\lambda_p}$	$\rho = P(\lambda \le \lambda_U)$							
	.67		.75		.90		.99	
	a	V_U	a	V_U	a	V_U	a	V_U
1.50	1.00	1.10	2.50	3.30	9.0	12.8	29.0	43.0
1.75	.80	.90	1.50	2.10	5.7	8.7	14.0	24.0
2.00	.70	.80	1.00	1.40	3.0	5.3	9.0	21.0
2.50	.60	.60	.70	.92	1.7	3.4	4.9	11.5
3.00	.45	.45	.50	.65	1.3	2.6	3.2	8.8
4.00	.37	.30	.40	.50	1.2	2.5	1.5	5.7
5.00	.33	.19	.35	.40	.7	2.3	1.3	5.5
7.50	.30	.18	.32	.35	.5	2.2	.9	4.6
10.00	.29	.17	.30	.33	.4	2.1	.7	4.0

Table II

Values for a if λ_p Assigned Mean

$\psi = \frac{\lambda_U}{\lambda_p}$	$\rho = P(\lambda \le \lambda_U)$			
	.67	.75	.90	.99
1.50	-	.90	7.0	28.0
1.75	-	.65	3.0	13.0
2.00	-	-	2.0	8.0
2.50	-	-	0.9	4.2
3.00	-	-	0.6	2.5
4.00	-	-	0.3	1.2
5.00	-	-	-	0.9
7.50	-	-	-	0.5
10.00	-	-	-	0.2

be off by a factor of m" then λ_p and m can be used to compute a and b as follows:

$$\lambda_p = \frac{a}{b}$$

$$\sigma = m\,\lambda_p = \sqrt{\frac{a}{b^2}} \qquad (4\text{-}10)$$

from which

$$a = \frac{1}{m^2}$$

$$b = \frac{1}{m^2\,\lambda_p} \qquad (4\text{-}11)$$

Also from equation (4-10) it is noted that m is the ratio of the standard deviation to the mean which is defined to be the coefficient of variation. To assess the impact that the choice of m will have on the final decision, the mean of the posterior must be examined. The revised estimate of the failure rate λ_p may be written

$$\lambda_p = \frac{a + f}{b + f} = \frac{f + \frac{1}{m^2}}{t + \frac{1}{m^2\,\lambda_p}} \qquad (4\text{-}12)$$

Thus, if there is no uncertainty associated with λ_p then $m = 0$ reflects total confidence. In this case $\lambda_p = \lambda_p$ irrespective of test data. Conversely, $m = \infty$ corresponds to total ignorance and $\lambda_p = f/t$, which is the best estimate obtained from test data only. Babillis and Smith have stated:

> For the time being the Gamma prior is being applied with either m = 0.75 or m = 1.75 depending on whether the test data reflects no failures or some failures respectively. General experience with this system has been favorable, and development efforts have been planned to provide substantiation and or refinement of the approach. (Ref 6:361).

Also, Feduccia reports that based on data dealing with observed vs predicted MTBFs of over one hundred ground electronic equipment and systems, the value of m was found equal to 1.38. If similar data exists for other types of equipment, then a more meaningful value for m can be determined from the sample means and standard deviation.

Equivalent Test Time Method. The last technique to be discussed in this paper addresses the predetermined contribution that a predicted value will have in the final decision. The basic approach reflected in this method is to relate the uncertainty associated with the prediction with the amount of usage experience likely to be encountered before the final decision. If one has high confidence in initial predictions, he has less need for additional experimentation. Conversely, if predictions are considered somewhat inaccurate, then greater reliance on test results is in order. Usually, the time associated with a test program is either specified or can easily be estimated. Then any amount of "pseudo-time" can be assigned to be compatible with a desired extent of contribution to the total time on which the final decision will be based. For example, if a 10,000 hour test program is expected and the decision to be made is to be based on 90% test results and 10% predictions then b = 1000 would be an appropriate assignment.

If the preselected portion of expected test time, b, is combined with λ_p, a unique Gamma distribution can be defined. The obvious need is a relationship between prediction uncertainty and the extent that the prediction should contribute to the posterior estimate. There are no established guidelines on this issue, however, it is

suggested that after careful and considered thought, a relationship could be devised for a particular program. For illustrative purposes in the next section, a hypothetical relationship is given in Table III below.

TABLE III

Equivalent Test Time for Priors

Degree of Belief in Predicted Value	Percent Contribution of Prior Parameter b	Divisor for Test Time T
0.99	0.75	0.33
0.90	0.50	1.00
0.75	0.25	3.00
0.67	0.17	5.00

In addition to time dependent reliability problems for which Gamma distributions apply, there are cases when time is not a factor. In these situations, it has been shown that distributions from the Beta family meet the previously stated criteria for priors.

<u>Assigning Beta Parameters</u>

As with the Gamma family, a Beta distribution is also defined by two parameters and thus a wide range of priors is possible. The assigned parameters combine readily with Binomial sampling statistics (the Binomial process is an experiment which observes s successes in n trials). If a Beta prior is updated with Binomial data, the resultant posterior is also a Beta distribution. In this case, if the success ratio p is assumed to be a random variable, then

$$\beta(p;\phi,\eta) = \frac{(\eta-1)!\,p^{\phi-1}\,(1-p)^{\eta-\phi-1}}{(\phi-1)!\,(\eta-\phi-1)!} \qquad (4\text{-}13)$$

for $\quad 0 \leq p \leq 1$

$\phi, \eta > 0$

The probability of a particular outcome of the Binomial experiment given p, can be expressed as follows

$$b(s/n,p) = \frac{n!\,p^{s}(1-p)^{n-s}}{s!\,(n-s)!}$$

It can be shown that the posterior distribution, $P(p/s,n)$, is also Beta, $\beta(p;\phi+s,\eta+n)$. The mean and variance of the two distributions are:

Prior	Posterior
$p = \frac{\phi}{\eta}$	$p = \frac{\phi+s}{\eta+n}$
$\sigma^2 = \frac{\phi(\eta-\phi)}{\eta(\eta-1)}$	$\sigma^2 = \frac{(\phi+s)(\eta-\phi+n-s)}{(\eta+n)(\eta+n-1)}$

The same rationale used to assign a and b for a prior Gamma distribution can be used to select ϕ and η for a prior Beta distribution. This discussion on the Beta distribution is presented for comparison with the time dependent situation for completeness only. The example in the next section does not illustrate the practical application of this distribution.

V. Bayesian Reliability Assessment Example

Now that the necessary building blocks have been explained, they will be used to construct a framework for the periodic assessment of system reliability based on analytical predictions and results of only limited testing. The proposed scheme will be presented by way of a hypothetical example.

The Original Low-level Detector, OLD, system has been in service for the past twenty years. Because of break throughs in technology pertaining to electronic jamming devices, this system is now only marginally effective in tracking targets. Also, the OLD design is of vacuum tube vintage and has required increasing amounts of downtime for maintenance in recent years. Logistics costs to support this system are inordinate because of limited sources of supply, since most tube manufacturers now concentrate on the production of solid state devices. A modern more effective system is currently being developed to replace the obsolete OLD system.

The Network for Early Warning, NEW, system prototype has been assembled and factory testing is now in progress.. Because of the urgent need for the NEW system, the development and acquisition contract was structured for concurrency. The terms and conditions of the contract stipulate that the release of funds for production will be predicated on satisfactory demonstration of system performance capability. The procuring agency and contractor have agreed on the portion of the total test program which must be accomplished prior to

production release. It was decided that the formal reliability test required to verify achievement of the contractual quantitative requirement could be deferred until after production commitment. However, an assessment of the inherent reliability designed into the NEW system must be made to determine if a minimal acceptable level has been achieved. If a threshold value cannot be reached, it will be necessary to redefine the system requirements and initiate a redevelopment effort for an alternative replacement for the OLD system. This contingency system will be designated XYZ. Now that the foundation has been laid for this case study, the remaining presentation will adhere to the established blueprint for solving decision problems.

Definition of the Problem

The primary objective of the designated procuring activity is the timely introduction of a cost-effective means of satisfying the operational need for sufficient warning of an advancing aggressor. Although the NEW system is based on existing technology to a great extent, it does contain some innovative features which constitute technical risks. There are many component types in the NEW system which are state-of-the-art devices and thus have questionable reliability characteristics. Although there are other uncertainties associated with the NEW development effort, it will be assumed that they will be addressed separately. Thus the problem will be defined: "From a reliability viewpoint, determine the advisability of committing funds for production of the NEW system." Thus A is the set of actions that a decision maker might take regarding the release of production funds. For illustrative purposes, the following choices are assumed:

a_1 = Release the funds; proceed into production.

a_2 = Wait three months for results of performance testing in progress.

a_3 = Wait six months for results of formal reliability test.

a_4 = Cancel program; initiate development of the contingency system.

To evaluate these options, the technical risks associated with NEW system reliability must be evaluated.

Initial Assessment of Uncertainties

The unknown quantity in this decision situation is the true system reliability. So in this case specific values for the system failure rate, λ , will constitute the possible states of nature. The contractual requirement for the NEW system is 10,000 failures per million hours, fpmh. To assess the probability that the inherent reliability is less than, equal to, or greater than this specified value, it is necessary to calculate a point estimate prediction and address the variability associated with it. At the critical design review for the NEW system, the information presented in Tables IV and V was presented by the contractor.

Table IV

NEW System Reliability Predictions

Item	Unit/Subsystem	Failure Rates $(X10^{-6})$		Pr $(\lambda \leq \lambda_U)$
		λ_{Pi}	λ_{Ui}	ρ
A	Array Antenna	2500	10000	.75
B	Beam Selector/Steerer	750	3000	.90
C	Control/Display Console	700	2100	.90
D	Data Processor	300	600	.99
E	Emitter/Detector	600	3000	.75
F	Frequency Randomizer	500	5000	.90

Table V

Parameters for a System Prior Distribution

Item	Qty Per Sys n_i	a_i	$b_i = \frac{V_{Ui}}{\lambda_{Ui}}$	$\mu_i = \frac{a_i}{b_i}$ $(X10^{-6})$	$n_i\mu_i$ $(X10^{-6})$	$\sigma_i^2 = \frac{n_i\mu_i}{b_i}$ $(X10^{-8})$
A	1	.40	50	8000	8000	16000
B	2	1.20	835	1435	2870	344
C	5	1.30	1240	1050	5240	423
D	1	9.00	29100	310	310	1
E	2	.35	133	2630	5260	3957
F	1	.40	420	950	950	227
					22,630	20,952

These tabulated data can be used to determine estimates for the system failure rate mean value, $\widehat{\mu_s}$, and any upper bound, $\widehat{\lambda_{su}}$, from the following relationships,

$$\widehat{\mu_s} = \sum_{i=A}^{F} n_i \mu_i \tag{5-1}$$

$$\widehat{\sigma_s^2} = \sum_{i=A}^{F} \sigma^2 \tag{5-2}$$

$$\widehat{\lambda_{su}} = \frac{2\widehat{\mu_s} + \frac{Z^2 \widehat{\sigma_s^2}}{\widehat{\mu_s}} + \sqrt{4Z^2 \widehat{\sigma_s^2} + \frac{Z^4 \widehat{\sigma_s^4}}{\widehat{\mu_s^2}}}}{2} \tag{5-3}$$

where Z = Normal deviate for the one sided confidence level of interest (e.g., for $\upsilon = .90$, $Z = 1.282$)

However, in decision problems there are usually several ranges of interest instead of just two (i.e., the intervals of $0 < \lambda < \widehat{\lambda_{su}}$ and $\widehat{\lambda_{su}} < \lambda < \infty$). In this example there will be six $P(\theta_j) = P(\lambda_j)$ divisions,

$$0 < \lambda_1 \leq 0.003$$
$$0.003 < \lambda_2 \leq 0.005$$
$$0.005 < \lambda_3 \leq 0.015$$
$$0.015 < \lambda_4 \leq 0.045$$
$$0.045 < \lambda_5 \leq 0.055$$
$$0.055 < \lambda_6 < \infty$$

To determine the $P(\lambda_j)$ it is convenient for later comparison to assume that the system failure rate distribution is also Gamma. A Gamma distribution can be transformed to a Chi-squared, χ^2, distribution with d degrees of freedom by letting $d = 2(a + 1)$ and $\chi^2(d) = 2b$. (Ref 153:31). In this case the values a_s and b_s for the system failure rate distribution can be determined from $\widehat{\mu_s}$ and $\widehat{\sigma_s^2}$, since

$a_s = \hat{\mu}_s^2 / \hat{\sigma}_s^2$ and $b_s = \hat{\mu}_s / \hat{\sigma}_s^2$. Then χ^2 tables can be used to find $P(\chi_j^2, d) = P(\lambda \le \lambda_U)$. The $P(\lambda_j)$ can be determined from the relationships $P(\lambda_j) = P(\chi_{j-1}^2, d)$ and $P(\chi_0^2, d) = 0$. For example, using equations (5-1) and (5-2) and the data listed in Table V, $P(\lambda_3)$ can be computed as follows,

$$a_s = \frac{\hat{\mu}_n^2}{\hat{\sigma}_s^2}$$

$$= \frac{(.022630)^2}{.00020952}$$

$$= 2.46$$

$$b_s = \frac{\hat{\mu}_s}{\hat{\sigma}_s^2}$$

$$= \frac{(.022630)}{.00020952}$$

$$= 108$$

$$\chi_2^2 = 2b_s \lambda_{U2}$$

$$= 2(108)(.005)$$

$$= 1.08$$

$$P(\chi_2^2, d) = P(1.08, 2(2.46) + 2)$$

$$= P(1.08, 6.92)$$

$$= .0070$$

$$\chi_3^2 = 2(108)(.015)$$

$$= 3.24$$

$$P(\chi_3^2, d) = P(3.24, 6.92)$$

$$= .1331$$

$$P(\lambda_3) = P(\chi_3^2, d) - P(\chi_2^2, d)$$
$$= .1381 - .0070$$
$$= .1311$$

The above sequence was repeated for the other $P(\lambda_j)$ and the results are listed in Table VI along with assigned point values, λa_j, for each interval which will be used in subsequent calculations.

Table VI

Values for $P(\lambda_j)$ Based on Predictions

j	$\chi_j^2 = 2b_s \lambda_{uj}$	$P(\chi_j^2, 2a_s + 2)$	$P(\lambda_j)$	λa_j
1	.65	.0014	.0014	.002
2	1.08	.0070	.0056	.004
3	3.24	.1381	.1311	.010
4	9.72	.7936	.6555	.030
5	11.88	.8959	.1023	.050
6	∞	1.0000	.1041	.060

Comparison of Consequences

Now that all prior knowledge concerning NEW system reliability has been quantified, the impact of the various λ_j on each of the a_i can be determined. In this case, the primary concern is to minimize anticipated life cycle costs since it has been assumed that the performance effectiveness of the NEW system is acceptable. There are many ways to estimate life cycle costs, most of which are rather elaborate and time consuming if any reasonable degree of precision is desired. The essential elements consist of development and acquisition costs and operation and maintenance, O&M, expenses. The

factors which relate to these basic costs for each of the systems in question are provided in Table VII. The most significant portion of total life cost is the maintenance expense. Table VIII contains the costs of monthly maintenance (assuming $5000 per repair) for each assigned failure rate, λa_j, and various system quantities.

Table VII

Life Cycle Cost Factors

Factor \ System	OLD	NEW	XYZ
Prototype Costs, P	-	\$ 50M	\$ 75M
Price per System, A	-	\$ 2M	\$2.5M
Delay Costs/Quarter, D	-	\$ 1M	-
Cancellation Costs, C_j	-	\$0-75M *	
O&M (Cost per Repair), R	\$ 5K	\$ 5K	\$ 5K
Observed/Specified λ	.05	.01	
Number of Systems, N	60	60	60
Delivery Rate (Systems Per Month)	-	5	5
Development Time (Months)	-	0	24
Production Lead Time (Months)	-	15	12

* C_1 = 75M; C_2 = 50M; C_3 = 25M; $C_4 = C_5 = C_6 = 0$

Table VIII

Monthly Maintenance Expense, R(N,λ) (In the millions of dollars)

Num of Sys N	System Failure Rate λ					
	.002	.004	.01	.03	.05	.06
5	.036	.072	.18	.54	.9	1.08
10	.072	.144	.36	1.08	1.8	2.16
15	.108	.216	.54	1.62	2.7	3.24
20	.144	.288	.72	2.16	3.6	4.32
25	.180	.360	.90	2.70	4.5	5.4
30	.216	.432	1.08	3.24	5.4	6.48
35	.252	.504	1.26	3.78	6.3	7.58
40	.288	.576	1.44	4.32	7.2	8.64
45	.324	.648	1.62	4.86	8.1	9.72
50	.360	.720	1.80	5.40	9.0	10.80
55	.396	.792	1.98	5.94	9.9	11.88
60	.432	.864	2.16	6.48	10.8	12.96

Another important consideration in this particular case is the impact that additional time has on effecting reliability improvements. Potential enhancement of reliability is dependent on many factors, including current technology, available resources, physical space, etc. There is no universal reliability growth model that applies to all types of equipment in any stage of development. For illustrative purposes, a simple linear relationship of estimated reliability improvement versus time is considered sufficient. To account for the additional time advantage in subsequent calculations, an estimated reliability improvement factor, $I(M, \lambda)$ will be used. Values for $I(M, \lambda)$ for $M = 0\text{-}30$ months and $\lambda = .004, .01, .03, .05$ and $.06$ can be obtained from Figure 1. For simplification, $I(M, .002) = 1$.

Now that all the pertinent information and relationships regarding individual cost elements for both existing and planned systems have been discussed, the expressions for computing the C_{ij} can be presented. Specifically, the C_{ij} represents total life cycle costs for a ten-year period commencing with scheduled production release. The delivery rates are such that a mix of existing and replacement occurs only during one year starting with the first increment. During this year, there is an average of 30 each present and replacement systems. For the alternative decisions, the corresponding consequences are:

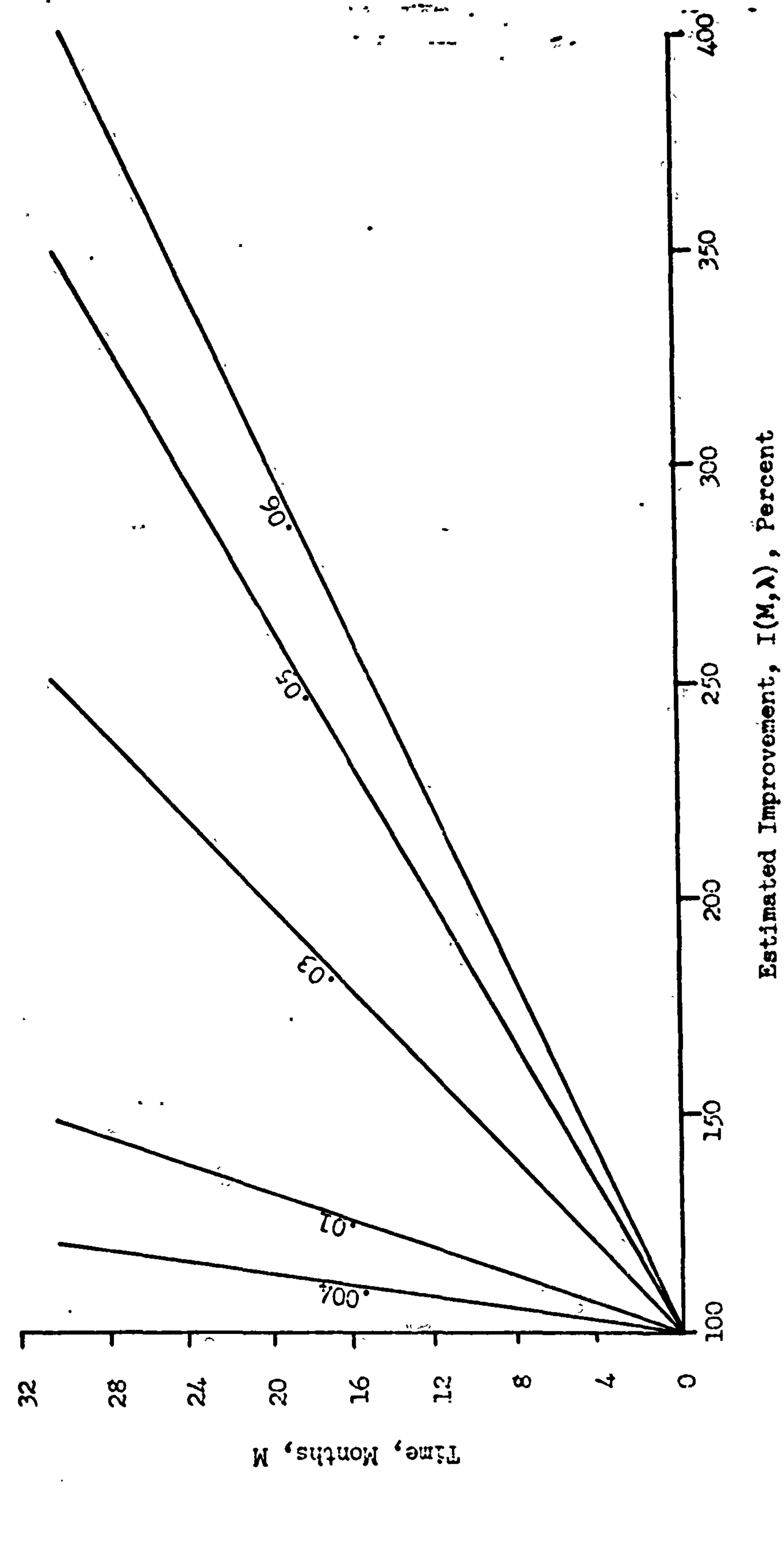

Figure 2. Reliability Growth Curves for Initial λ = .004, .01, .03, .05 and .06

$$C_{1j} = Pn + 60An + 15R(60, .05) + 12R(30, .05) + 12R(30, \lambda_{aj}) + 105R(60, \lambda_{aj}) \tag{5-4}$$

$$C_{2j} = Pn + 60An + D + 18R(60, .05) + 12R(30, .05) + \left[\frac{12R(30, \lambda_{aj}) + 102R(60, \lambda_{aj})}{I(3, \lambda_{aj})}\right] \tag{5-5}$$

$$C_{3j} = Pn + 60An + 2D + 21R(60, .05) + 12R(30, .05) + \left[\frac{12R(30, \lambda_{aj}) + 99R(60, \lambda_{aj})}{I(6, \lambda_{aj})}\right] \tag{5-6}$$

$$C_{4j} = Pn + Px + 60Ax + 36R(60, .05) + 12R(30, .05) + \left[\frac{12R(30, \lambda_{aj}) + 84R(60, \lambda_{aj})}{I(24, \lambda_{aj})}\right] \tag{5-7}$$

Thus, from equation (5-6), C_{33} (in millions of dollars) can be determined as follows,

$$C_{33} = 50 + 120 + 2 + 21(10.8) + 12(5.4) + \left[\frac{12(1.08) + 84(2.16)}{1.1}\right]$$

$$= 670$$

The remaining C_{ij} were calculated in a similar manner and the results are listed in Table IX. Since the C_{ij} are expressed in quantitative terms, assignment of utility measures is unnecessary.

Table IX

Production Release Decision Matrix

(C_{ij} in millions of dollars)

Decision \ System Failure Rate	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
a_1 Proceed	445	493	624	1116	1596	1836
a_2 Wait 3 mos.	477	521	652	1039	1363	1498
a_3 Wait 6 mos.	509	551	670	987	1220	1315
a_4 Cancel	843	846	893	994	1053	1072

Calculation of Expected Value

The solution to this production release decision problem is to take the action, a_i, which is most likely to result in the lowest life cycle cost. This selection can be made by comparing the expected value of each alternative, $E(a_i)$. The $E(a_i)$ can be computed from the data in Table IX by using equation (2-1),

For example,

$$E(a_1) = \sum_{j=A}^{F} C_{1j}\, P(\lambda_j)$$

$$= (445)(.0014) + (493)(.0056) +$$

$$(624)(.1311) + (1116)(.6555) +$$

$$(1596)(.1023) + (1836)(.1041)$$

$$= \$1171M \qquad (5\text{-}9)$$

Similarly, the other $E(a_i)$ were computed and the following values were obtained for $E(a_2)$, $E(a_3)$, $E(a_4)$ respectively: \$1069M, \$1000M, \$994M. Thus, the logical decision based on predicted reliability data only is a_4 or cancel the NEW program and initiate procurement of the XYZ system.

Second Assessment of Uncertainties

However, in addition to these analytical predictions, logs of individual unit/subsystem operating times and failures have been maintained as portions of the NEW system have progressed through various phases of testing. The specific information reported is listed in Table X.

Table X

Cumulative Unit/Subsystem Test Data

Unit/Subsystem	A	B	C	D	E	F
Total Failures f_i	5	1	4	0	1	1
Total Time t_i	1500	2000	4750	900	2400	1500

These data also reflect a measure of system reliability and may be used independent of the prior information to determine estimates for $P(\lambda_j)$. Again the χ^2 distribution may be used by letting $d = 2(f + 1)$ and $\chi_j^2(d) \doteq 2t\lambda_{uj}$. If it is assumed that the data in Table X represent $f = 12$ failures in approximately $t = 1000$ equivalent hours of system operation, then the $P(\lambda_j)$ can be determined using the same procedure as before. These $P(\lambda_j)$ - based solely on the observed test data - are listed in Table XI. When these probabilities are used in the decision matrix (Table IX), the values for $E(a_i)$ are: \$755M, \$755M, \$755M, and \$920M. Therefore the test results are inconclusive as far as a definite decision is concerned.

Table XI

Values for $P(\lambda_j)$ Based on Test Data

j	$\chi_j^2 = 2t\lambda_{uj}$	$P(\chi_j^2, 2f + 2)$	$P(\lambda_j)$	λa_j
1	6	.0000	.0000	.002
2	10	.0020	.0020	.004
3	30	.7324	.7304	.010
4	90	1.0000	.2676	.030
5	110	1.0000	.0000	.050
6	∞	1.0000	.0000	.060

Final Assessment of Uncertainties

The test data may also be combined with the prior information recorded in Tables IV and V to produce a more accurate revised system failure rate distribution. The parameters and statistics of the

posterior distributions will be distinguished from prior values by using script letters and bold type. The posterior parameters for the unit/subsystem failure rate distributions are $A_i = a_i + f_i$ and $B_i = b_i + t_i$. The posterior system failure rate distribution mean and variance are $\boldsymbol{\mu}_s$ and $\boldsymbol{\sigma}_s^2$, respectively. In this particular case, when the individual unit/subsystem "pseudo" and actual failures and operating times are combined, the resultant values for A_i and B_i are those listed in Table XII. The values for $\boldsymbol{\mu}_s$ and $\boldsymbol{\sigma}_s^2$ are determined by following the same procedure used to obtain μ_s and σ_s^2. Likewise, the system Gamma parameters, A_s and B_s, can be computed the same way as before. These quantities may then be used to determine the individual $P(\lambda_j)$ from expressions associated with Bayesian one sided upper confidence limits given by Nagy (Ref 153:29).

$$\begin{aligned} \chi_j^2\ (d) &= 2B_s\ \lambda_{uj} \\ d &= 2A_s \end{aligned} \tag{5-10}$$

Again, since the λ_{uj} are stipulated, the values for χ_j^2 can be calculated and the χ^2 tables can be used to find $P(\chi_j^2, 2A_s)$. The values for $P(\lambda_j)$ which result from these computations are listed in Table XIII. When these revised estimates are used in equation (5-8) to compute the $E(a_i)$ the results (in millions of dollars) are: 652, 674, 688, and 898. Thus, the preferred course of action is clearly a, and production release may be granted without delay. Note that this decision is the reasonable choice although $\boldsymbol{\mu}_s$ is greater than the failure rate specified in the contract. In this case, it has been illustrated that the planned deployment of the NEW system with an estimated reliability of approximately only 87% the required level.

is more cost effective than the continued use of the OLD system or the three year delay required for the XYZ system.

Table XII

Parameters for a System Posterior Distribution

Item	n_i	A_i	B_i	$\phi_i = \frac{A_i}{B_i}$ (X 10^{-6})	$n_i \phi_i$ (X 10^{-6})	$\sigma_i^2 = \frac{n_i \phi_i}{B_i}$ (X 10^{-8})
A	1	5.40	1550	3484	3484	224.8
B	2	2.20	2835	776	1552	54.7
C	5	5.30	5990	885	4425	74.0
D	1	9.00	30000	300	300	1.0
E	2	1.35	2533	533	1066	42.1
F	1	1.40	1920	729	729	38.0
					11,556	434.6

Table XIII

Values for $P(\lambda_j)$ Based on Combined Inputs

j	$\chi_j^2 = 2B_s \lambda_{uj}$	$P(\chi_j^2, 2A_s)$	$P(\lambda_j)$	λ_{a_j}
1	15.95	.0000	.0000	.002
2	26.59	.0001	.0001	.004
3	79.77	.9425	.9424	.010
4	239.31	1.0000	.0575	.030
5	292.49	1.0000	.0000	.050
6	∞	1.0000	.0000	.060

VI. Assessment Technique Analysis and Refinement

The sample information used in the example presented was obviously selected to illustrate a point. There is absolutely no guarantee that the pooling of reliability predictions and test data will always result in such a drastic change in the preferred course of action. However, the scheme outlined does produce better informed decisions which are normally less costly because less experimentation is usually required. The reason for this is better appreciated if the contributions of the two inputs are analyzed.

Examination of Input Contributions

As mentioned in Section III, the quantification of prior information (in this case reliability predictions) narrows the range of exploration for the true state of nature (system failure rate). The outcomes of experimentation (failure and time data) provide a further reduction to the area of consideration by decreasing the variability of the initial estimates. These contributions are apparent when the unit/subsystem and system failure rate densities are examined. In the example, the greatest amount of uncertainty was contributed by items A and E. The prior and posterior failure rate density functions for these two items are plotted in Figures 3 and 4 respectively. In each instance, the posterior variance is considerably smaller than the prior variance. The magnitudes of difference can be determined by comparing the values for σ_j^2 and ϕ_j^2 (listed in Tables V and XII) for these two and the other items. (Also shown in Figures 3 and 4 are $g(\lambda; A,B)$ for other selected test data for comparative purposes.)

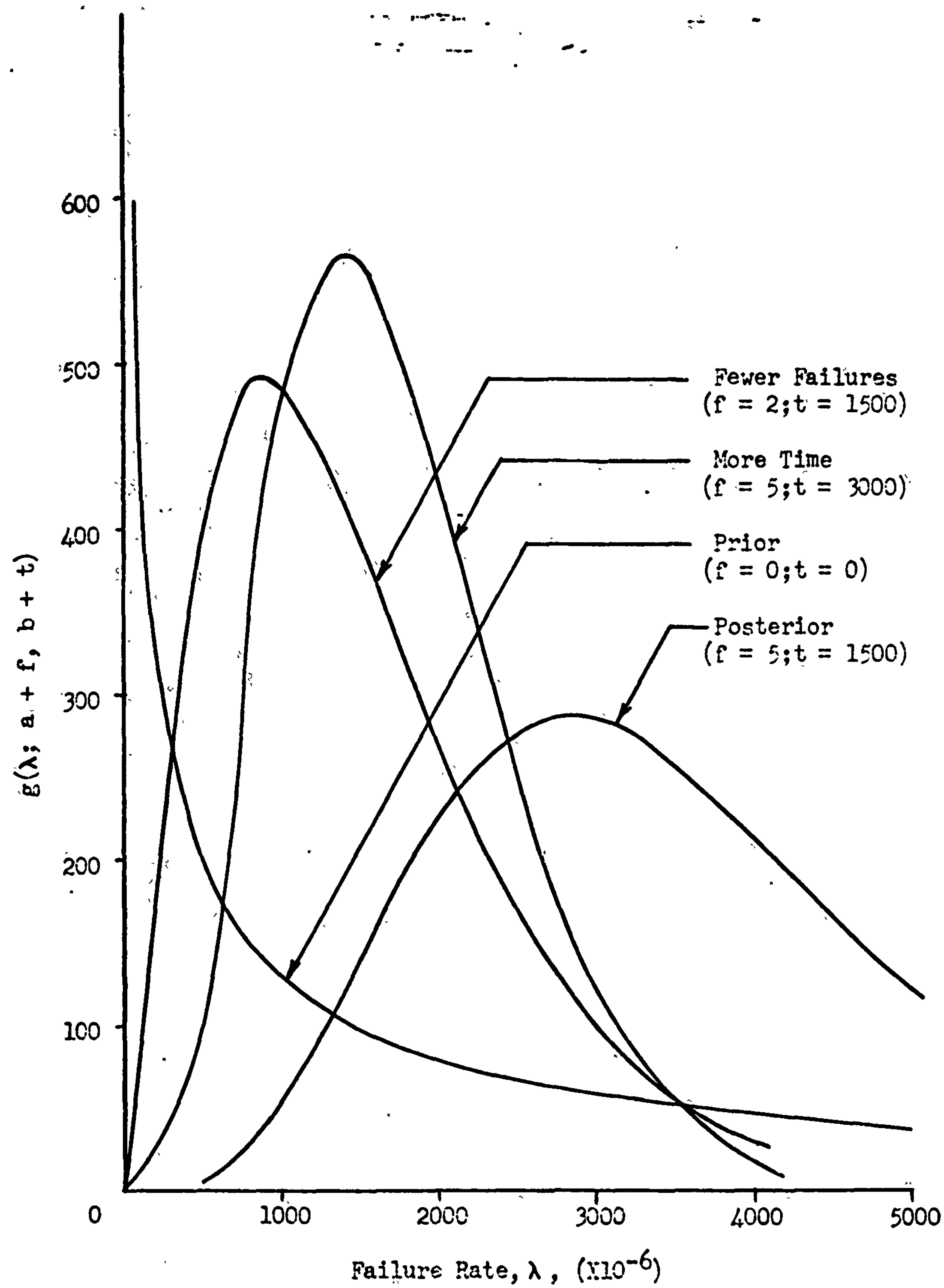

Figure 3. Effect of Test Data on Antenna Array Gamma Distribution

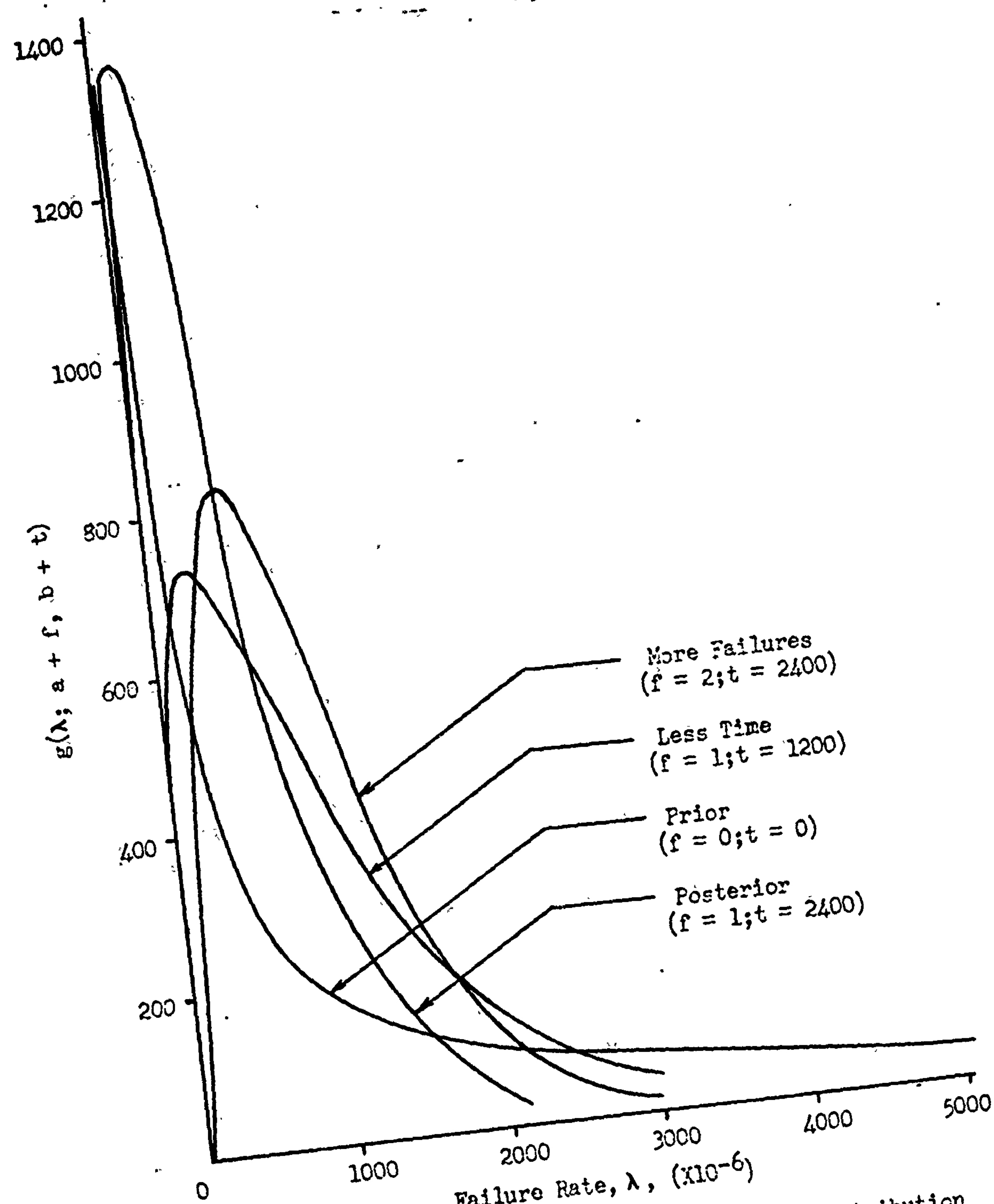

Figure 4. Effect of Test Data on Emitter/Detector Gamma Distribution

The values of σ_s^2 and σ_s^2 may also be found in Tables V and XII. These quantities dictate the shapes of the prior and posterior system failure rate densities. Again the posterior variance is less than the prior, and therefore, the range of the true system failure rate is smaller which results in a more accurate estimate of reliability. Another factor which influences the accuracy of the $P(\lambda_j)$ quantities is the assumed system distribution.

<u>Gamma Vs Normal System Distributions</u>

As previously stated, a Gamma system failure rate distribution was assumed as a matter of convenience. Another statistical distribution which is widely known and easily applied is the Normal distribution. However, in order to employ the Normal distribution two parameters must be specified or estimated. To analyze the sensitivity of the assessment technique used in the example, four additional sets of sample information were assumed and resultant choices compared with original decisions. Rather than assuming a particular value for the standard deviation of the hypothetical test data, it was decided to create sufficient information from which a sample standard deviation could be computed. The additional sets of information include two unfavorable sets and two favorable sets as compared with the original example set. The unfavorable data include both the case in which the same number of failures are observed in half the time and the case in which twice the failures are observed in the same amount of time. The favorable data include both the case in which half the number of failures are observed in the same amount of time and the case in which the same number of failures are observed in twice the time. The

sample failure information for these four new cases plus the original case is presented in Table XIV.

For each set of data, a set of $P(\lambda_j)$ was computed twice - once assuming a Gamma system distribution and again assuming a Normal distribution. Then the Normal distribution was assumed, the sample μ_s and σ_s^2 were calculated from the following relationships,

$$\mu_s = \frac{1}{m} \sum_{j=1}^{m} \frac{1}{t_j}$$

$$\sigma_s^2 = \frac{\sum_{j=1}^{m} (1/t_j - \mu_s)^2}{m - 1} \qquad (6\text{-}1)$$

For example, using the data in Table XIV in the first and third columns,

$$\mu_s(f,.5t) = \frac{1}{12}\left[\frac{2}{35} + \frac{5}{40} + \frac{4}{45} + \frac{1}{50}\right]$$

$$= .024253$$

$$\sigma_s^2(f,.5t) = \frac{1}{11}\Big[2(.028571 - .024253)^2 + 5(.025000 - .024253)^2$$

$$+ 4(.022222 - .024253)^2 + (.020000 - .024253)^2\Big]$$

$$\doteq .002605$$

To obtain the $P(\lambda_j)$, it is necessary to compute $Z_j = (\lambda_{uj} - \mu_s)/\sigma_s$ and then use standard statistical tables to obtain $P(Z_j) = P(\lambda \leq \lambda_{uj})$. The $P(\lambda_j)$ can than be determined from the relationships $P(\lambda_j) = P(Z_j) - P(Z_{j-1})$ and $P(Z_o) = 0$.

The results of recomputing the $E(a_i)$ for each set of data are listed in Tables XV and XVI when Gamma and Normal distributions, respectively, are assumed to apply. Analysis of these results indicates that the assessment technique yields the same decision for

Table XIV

Sample Failure Information

Time Between Failure (hrs)	Number of Failures				
	Example Case (f,t)	Unfavorable Data (f,.5t)	Unfavorable Data (2f,t)	Favorable Data (.5f,t)	Favorable Data (f,2t)
25			1		
30			2		
35		2	3		
40		5	8		
45		4	5		
50		1	3		
55			1		
60	1		1		
70	2				
75	1				
80	1				
85	1				
90	3				
95	2				
100	1				
135				1	1
150				1	1
155				1	2
165					3
170					1
175				1	1
180				1	2
205				1	1

Table XV

$E(a_i)$ Values Assuming Gamma System Distribution

$P(\lambda_j)$ Data Source	$E(a_1)$	$E(a_2)$	$E(a_3)$	$E(a_4)$
Prior Information (Predictions)	1171	1069	1000	994*
Unfavorable Test Data (12/500)	1101	1027	976*	990
Unfavorable Test Data (24/1000)	1111	1039	934*	993
Example Case Test Data (12/1000)	755*	755*	755*	920
Favorable Test Data (6/1000)	595*	622	643	950
Favorable Test Data (12/2000)	632*	662	685	950
Unfavorable Posterior (12/500)	1081	1011	964*	988
Unfavorable Posterior (24/1000)	1116	1039	937*	994
Example Case Posterior (12/1000)	652*	674	638	899
Favorable Posterior (6/1000)	617*	645	664	976
Favorable Posterior (12/2000)	650*	680	702	962

*Minimum $E(a_i)$

Table XVI

$E(a_i)$ Values Assuming Normal System Distribution

$P(\lambda_j)$ Data Source	$E(a_1)$	$E(a_2)$	$E(a_3)$	$E(a_4)$
Prior Information (Predictions)	983	926	891*	962
Unfavorable Test Data (12/500)	1116	1039	987*	994
Unfavorable Test Data (24/1000)	1098	1024	975*	990
Example Case Test Data (12/1000)	704*	711	716	907
Favorable Case Test Data (6/1000)	610*	638	658	888
Favorable Case Test Data (12/2000)	661*	680	692	900
Unfavorable Posterior (12/500)	1079	1010	963*	926
Unfavorable Posterior (24/1000)	1092	1020	972*	989
Example Case Posterior (12/1000)	648*	671	685	898
Favorable Posterior (6/1000)	615*	643	662	890
Favorable Posterior (12/2000)	619*	647	665	[illegible]

*Minimum $E(a_i)$

either distribution except when only the prior information is used to compute the $P(\lambda_j)$. Also the extent of discrimination between the $E(a_i)$ computed from the same data base appears to be remarkably similar for the two assumed distributions. Comparison of the $E(a_i)$ based on unfavorable vs favorable test results reveals that the intuitive decrease in calculated value occurs as fewer failures are observed in greater time. Overall, the assessment technique seems realistic and practical.

Other Practical Considerations

In addition to the features of the technique presented with the example and discussed above, there are two other refinements which are considered worthy of discussion. One pertains to the consideration of different opinions when assigning parameters of the unit/subsystem prior failure rate distributions. The other relates to the treatment of different types of test data when determining operating times.

Weighting Prior Assignments. There are many instances in which different individuals provide inputs for the various pieces of equipment that make up a system. The analyst might also wish to combine the inputs from several sources in order to formulate a single overall estimate for an item failure rate. Fox advances the idea of assigning weighting factors to each contributor (Ref 84:3). These weights can be based on either the contributors accuracy and consistency of previous predictions or his extent of participation if a joint assessment is required (i.e., a prior based on combined government, consultant, contractor inputs). In either case, if a weighting factor ϕ is assigned to the k^{th} of r individuals, then the parameters for a

Gamma prior can be computed from the following expressions,

$$a = \sum_{k=1}^{r} \phi_k a_k$$

$$b = \sum_{k=1}^{r} \phi_k b_k$$

Weighting Heterogeneous Test Data. The other weighting factor considered important is a multiplier which accounts for the difference in severity of the various types of tests to which an item is usually subjected. The concept of attaching more significance to data obtained under more difficult environments has been proposed by Pozner. He states this may be done "by weighting the time experience in particular environments by the k factor corresponding to these environments. These k factors are environmental failure rate acceleration factors such as those in MIL-STD-756." (Ref 162:139). Test severity weights need not be of the magnitude generally associated with acceleration factors. The important consideration is that the value computed or assigned reflect the extent of additional exposure experienced by the equipment beyond that normally expected. As with the reliability predictions, test severity weights based on considered opinion or technical judgment of competent engineers may be used. Again, if several subjective inputs are to be combined, the contributors' weights can be included in deriving an overall set of test severity weights for the various types of tests to be performed. In general, a value k for a particular test can be determined from the following expression,

$$K = \sum_{j=1}^{r} \gamma_j k_j$$

where γ_j = contributors weight

k_j = individual test severity weight

r = number of contributors

Note that the term used for the contributor's weight is different than the one used in assigning priors. This was done purposefully to distinguish between judgements pertaining to reliability predictions and those concerning difficulty of test environments.

With the addition of these two weighting factors a recap of the revised assessment technique for the general case is in order.

<u>Revised General Approach</u>

The Bayesian reliability assessment technique may be implemented in a variety of situations. It is not necessary for the situations to be based on the need to make a particular decision as was illustrated. The procedures outlined and methods presented may also be used for periodic determination of the status of reliability achievement. In summary, the following step-by-step activities are required to obtain and update reliability estimates using subjective evaluation and Bayes techniques:

(1) Quantification of all prior knowledge concerning failure rate predictions. This effort may be a singular or combined input. If combined, contributor weights may be used to obtain single values for prior parameters for calculation purposes. The assertion of uncertainty must be carefully considered in light of potential

impact when combined with measured data. Parameters for prior distributions may be determined by a variety of methods and Tables I and II should be beneficial.

(2) Determination of a system prior distribution. This may be accomplished by summing the means and variances of constituent element failure rate distributions. If only a one-sided upper confidence limit for the system failure rate is of interest, equation (5-3) may be used. If discrete intervals are desired, then either a Gamma or Normal system distribution may be assumed and particular probabilities may be computed as illustrated.

(3) Collection of time and failure data. This is the most critical activity associated with any estimation task. Posterior estimates are only meaningful if they are based on accurate and complete information. In order to obtain high quality data, special emphasis must be placed on the proper training and motivation of personnel responsible for maintaining records. There is also the question of which types of anomalies to consider and which ones to censor. The definition of relevance is many times a subject for negotiation if the products of the assessment effort are used for acceptance purposes. Also, test times may be adjusted to reflect severity of equipment exposure. This should be done before tests are started to avoid undue bias based on outcomes.

(4) Determination of a system posterior distribution. This involves adding the constituent element "pseudo" failures and times with corresponding observed values to modify the prior parameters. System reliability indices can be computed from the posterior distribution in the same manner as from the prior distribution.

(5) Analysis of sensitivity. This optional task may be performed if there is doubt concerning the impact that a particular quantity has on the overall results. If more precision is desired than that achievable from assuming either a Gamma or Normal distribution, then Monte Carlo techniques can be employed to determine exact confidence bounds.

The tasks outlined above are quite general and are considered to be useful for widespread applications. To facilitate the recording of the inputs necessary for a Bayesian assessment effort, sample worksheets are included as Appendix B.

VII. Conclusions and Recommendations

Conclusions

It has been shown that Bayesian statistics used in conjunction with decision theory offer a suitable framework for solving cost effectiveness type problems involving uncertainty. When the uncertainty is reliability, there is considerable advantage to be realized in combining predictions with test data to obtain greater precision in the reliability estimate. Therefore, it is concluded that the implementation of Bayesian techniques in the solution of reliability decision problems can produce more conclusive results with the added advantages of being economically practical and intuitively appealing.

Recommendations

Based on the findings of this study, it is recommended that the reader desiring to further pursue the field of Bayesian statistics consider the following topics:

(1) Development of simple algorithms and other aids to establish prior distributions based on predictions.

(2) Investigation of other closed distribution forms to fit the system failure rate.

(3) Development of a simple generalized Monte Carlo model which can be used to determine exact system failure rate bounds.

Also, it is suggested that this study be used as:

(1) an aid in the study of Bayesian statistics and decision theory;

(2) as a source of references and possible thesis topics for future Air Force Institute of Technology students.

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APPENDIX A

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APPENDIX B

BAYESIAN RELIABILITY ASSESSMENT

SAMPLE WORKSHEETS

Worksheet A

QUANTITATIVE PREDICTION/UNCERTAINTY DATA

System____________ Analyst____________

Item i	Unit/Subsystem		λ_p	Method 1		2	3	a	b
	Nomenclature	Part No.		λ_U	Pr	m	b		

Worksheet B

PRIOR KNOWLEDGE SUMMARY

System_______________ Unit/Subsystem________

Analyst	Weight ϕ	a	b	ϕ a	ϕ b
$a = \sum \phi a$	1.00	$b = \sum \phi b$	$\sum$		

Worksheet C

PRIOR DISTRIBUTION PARAMETER DATA

System ______________________

Item i	Unit/Subsystem		Qty/Sys n_i	a_i	$n_i a_i$	b_i	$\mu_i = \frac{n_i a_i}{b_i}$	$\sigma_i^2 = \frac{n_i a_i}{b_i^2}$
	Nomenclature	Part No.						
$\mu_s = \sum \mu_i$	$\sigma_s^2 = \sum \sigma_i^2$					$\sum$		

Worksheet D

TEST SEVERITY WEIGHTING FACTORS

System____________________

Analyst	Contributor Weight γ	Test 1		Test 2		Test 3		Test 4	
		k	γk	k	γk	k	γk	k	γk
$K = \sum \gamma k$ $\sum$	1.00								

Worksheet E

UNIT/SUBSYSTEM FAILURE RECORD

System________________ Unit/Subsystem____________________ Month________________

Type Test/ Data Source	Proc No	Weight K	Week 1		Week 2		Week 3		Week 4		Totals		
			t	f	t	f	t	f	t	f	t	Kt	f
λ_p =	$\lambda_p t_m$ =		$t_m = \sum Kt$				$f_m = \sum f$						

Worksheet F

SYSTEM FAILURE DATA SUMMARY

System____________________ Period________________

Item i	Unit/Subsystem		Previous Totals		Month 1		Month 2		Month 3		Current Totals	
	Nomenclature	Part No.	t_o	f_o	t_m	f_m	t_m	f_m	t_m	f_m	t_c	f_c

Worksheet G

POSTERIOR DISTRIBUTION PARAMETER DATA

System________________________

Item i	Unit/Subsystem		$A_i = a_i + f_c$	$B_i = b_i + t_c$	$\mu_i = \frac{nA_i}{B_i}$	$\sigma_i^2 = \frac{nA_i}{B_i}$
	Nomenclature	Part No.				
$\mu_s = \sum \mu_i$	$\sigma_s^2 = \sum \sigma_i^2$			$\sum$		

Vita

Lewis Ray White [redacted] Virginia, [redacted] which he enrolled at Virginia Polytechnic Institute, Blacksburg, Virginia. There he participated in the school's Cooperative Engineering Program under which he alternated his academic quarters with periods of work as a student trainee at Norfolk Naval Shipyard, Portsmouth, Virginia. In June 1961, he was graduated with the Degree of Bachelor of Science in Mechanical Engineering and a month later was commissioned a Second Lieutenant in the USAF Reserve. He was ordered to active duty in November 1961 and was assigned as a Project Engineer to the Air Force Plant Representative Office at the Convair Division of General Dynamics Corporation, San Diego, California before attending the Air Force Institute of Technology.

[redacted]

This thesis was typed by Miss Louise J. Houle